Mathematical Asides[Ab1]

1. Single Variable Normalized Gaussian Distribution

$f(x) \equiv \frac{1}{\sqrt{2\pi}\sigma} \mathrm{Exp}\left[-\frac{(x-\hat{x})^2}{2\sigma^2}\right]$	Mean = $\hat{x}$, Variance = σ^2, Skewness = 0, Excess = 0, FWHM $\cong 2.35482\,\sigma$

2. Two Variable Normalized Gaussian Distribution, -1 < r < 1

$f(x,y) \equiv \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \mathrm{Exp}\left[-\frac{1}{2(1-\rho^2)}\left\{\frac{x^2}{\sigma_x^2} - 2\rho\frac{xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right\}\right]$		
$\langle x^2 \rangle = \sigma_x^2,$	$\langle y^2 \rangle = \sigma_y^2,$	$\langle xy \rangle = \rho\sigma_x\sigma_y$

3. Gamma Function

$\Gamma(z) \equiv \int_0^\infty t^{z-1}e^{-t}dt$	$\Gamma(z+1) = z\Gamma(z);\ \ \Gamma(1-z) = \frac{\pi}{\Gamma(z)\sin\pi z}$ $\Gamma(n) = (n-1)!,\ n = \mathrm{int\,eger}$ $\Gamma(1/4) \cong 3.6256, \Gamma(1/3) \cong 2.6789, \Gamma(1/2) = \pi^{1/2}$

4. Dirac Delta Function Identities

$\lim_{\varepsilon \to 0^+} \frac{1}{\omega \mp i\varepsilon} = \mathrm{P.V.}\frac{1}{\omega} \pm i\pi\delta(\omega)$	$\delta(ax) = \frac{\delta(x)}{\lvert a \rvert}$
$\delta(x^2 - a^2) = \frac{\delta(x-a) + \delta(x+a)}{2\lvert a \rvert}$	$\delta(\phi(x)) = \sum_s \frac{\delta(x - x_s)}{\lvert \phi'(x_s) \rvert}$